Computational Issues of Neutron transport

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Summary of the main results

In the period May 30 – December 30, 2007 we studied various formulations of the equations of the neutron transport and corresponding numerical methods for their solution. These formulations allow better understanding the properties of numerical methods for solving the transport equation in the case of heterogeneous media, especially in the diffusion limit.

We have studied the least-squares method (Petrov-Galerkin) for the preconditioned equation (namely, multiplied by an operator R) that leads to variation formulation with a symmetric and positive definite bilinear form on a space. We show that the solution of the diffusion limit of transport equation is in fact the "elliptic projection" on H_0^1, a closed subspace of W, with respect to this form. This has an important implication: the standard Galerkin method for this formulation has very good approximation properties and produces the solution of the diffusion limit with a good accuracy. Next, we considered the difference between the exact solution and the diffusion limit corrected with the gradient of the solution to the diffusion limit. The existing estimate gives an error of order $O(\varepsilon)$ in a maximum-norm. We prove a new estimate in certain energy norm of order $O(\sqrt{\varepsilon})$.

Further, we have studied the mixed formulation of the neutron transport equation and have introduced the appropriate spaces for the corresponding finite element approximation. We also introduce the spaces W, where the solution is sought, and prove that this choice guarantees the validity of an *inf-sup* condition. Moreover, we show that the solution to the mixed formulation of the "diffusion limit" equation represents that "mixed elliptic" projection of the solution of the neutron equation in mixed form.

A detailed version of the results of the research could be found in the technical report [1].

References

[1] R.D. Lazarov, J.E. Pasciak, P.S. Vassilevski, and L.T. Zikatanov, Formulations of neutron transport equation that are well conditioned in the diffusion limit, Preprint, (in preparation).